ZIV-ZAKAI BOUND ON TIME DELAY ESTIMATION IN UNKNOWN CONVOLUTIVE RANDOM CHANNELS

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ABSTRACT

Using the Ziv-Zakai bound (ZZB) methodology, we develop a Bayesian (MSE) bound on time delay estimation (TDE) in a wideband convolutive random channel. The channel is modeled as a tapped delay line, whose taps are Gaussian random variables that may be non-zero mean and correlated. The derivation does not assume channel knowledge at the receiver, and so realistically takes into account the effects of the unknown channel when estimating time delay. The channel model can represent wideband and ultra-wideband, as well as line of sight and non-line of sight cases, through selection of model parameters. The bound development involves the minimum probability of decision error under a hypothesis test, and we show this can be equivalently formulated as a probability of detection error for binary pulse position modulation (PPM) signals. An expression for the error probability, and thus the ZZB on TDE, is derived using a moment generating function approach. Effects of system design and channel distribution parameters are studied through the ZZB.

1. INTRODUCTION

As a fundamental problem in signal processing, time delay estimation (TDE) performance bounds have been extensively studied, especially employing Cramér-Rao bounds (CRBs) with deterministic parametric models. Yau and Bresler developed CRBs for superimposed and delayed parameterized signals [1]; this approach is readily adapted to the case of TDE in a known deterministic multipath channel [2]. The resulting CRBs provide a local bound on TDE, but are not tight for all signal-to-noise ratios (SNRs), and do not account for random channel effects. In addition, in non-line of sight (NLOS) wideband channels, the CRB cannot account for bias arising from the lack of a direct LOS path between transmitter and receiver.

In this paper, we develop a Bayesian mean square error (MSE) bound on TDE under a random wideband (convolutive) channel model, using a Ziv-Zakai bound (ZZB) approach. The ZZB [3, 4, 5] for TDE uses a random prior on the time delay, and relaxes CRB-imposed regularity conditions on the waveform. The ZZB approach provides tight bounds over a large SNR range. A survey of TDE bounds in additive white Gaussian noise (AWGN) channels, or narrowband fading channels, is given in [6]. ZZBs on TDE have also been developed for multiple narrowband fading channels [7], and for ultra-wideband signals in AWGN channels [8].

In [9], an average ZZB under the wideband convolutive random channel case is developed. A conditional bound is first derived, conditioned on a given channel realization, and it is then averaged over the random channel model. The development incorporates a probability of bit detection error expression for an optimal receiver that has perfect knowledge of each channel realization. The resulting average ZZB, sometimes referred to as a perfect measurement based lower bound [10], reveals the impact of multipath fading as well as randomness over channel realizations that occur due to movement of the scatterers or the terminals.

Here, we present a ZZB on TDE under a convolutive random channel. We assume the receiver knows the channel distribution, but does not have knowledge of the channel realization. This represents a more realistic (and tighter) bound than that derived in [9]. Hereafter we refer to the bound developed in this paper as the ZZB. The binary decision error probability required by the ZZB is equivalent to the binary bit detection error probability, similar to the development in [9]. However, the decision statistic for the optimal log-likelihood ratio (LLR) test relies on the outputs of a series of correlators matched to multipath components. Their hypothesized distributions form a basis for a composite hypothesis test under nuisance channel parameters, as described by eq. (296) in [10]. In our problem, both the multipath channel taps and noise are treated as Gaussian random variables. The LLR is shown to follow a general quadratic form of a Gaussian random vector. We then find the probability density function (pdf) via a moment generating function (MGF) approach [11], that in turn leads to the minimum detection error probability expression that is needed to complete the ZZB derivation.

This paper is organized as follows. Section 2 briefly

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reviews the ZZB. In Section 3, the signal, channel, and receiver models are introduced. In Section 4, the composite hypothesis testing model is set up and the ZZB is developed under unknown channel realizations. Section 5 presents some bound results with different system parameters, and Section 6 concludes the paper.

2. REVIEW OF THE ZIV-ZAKAI BOUND

The development of the ZZB links estimation of time delay t_0 with a hypothesis testing problem that discriminates a signal at two possible delays [3, 4, 5]. For a received signal at one of the two possible delays $h(t-a)$ or $h(t-a-\Delta)$, where $\Delta > 0$ and $a, a+\Delta \in [0, T]$, a binary decision problem based on a delay estimate $\widehat{t_0}$ is described as follows

$$\begin{aligned} &\text{Decide } H_0: \quad t_0 = a \qquad\;\; \text{if } |\widehat{t_0}-a| < |\widehat{t_0}-a-\Delta|, \\ &\text{Decide } H_1: \quad t_0 = a+\Delta \;\; \text{if } |\widehat{t_0}-a| > |\widehat{t_0}-a-\Delta|. \end{aligned}$$

In general, the decision error is independent of a. Let $P_e(\Delta)$ be the minimal probability of error achieved by the optimum detection scheme in making a decision on the above hypothesis test. Then, the ZZB on TDE provides an MSE bound and takes the following form [3]

$$\overline{\epsilon^2} \geq \frac{1}{T}\int_0^T \Delta(T-\Delta)P_e(\Delta)d\Delta. \quad (1)$$

Evaluation of the bound (1) relies on finding the minimal probability of error $P_e(\Delta)$. Note that this is equivalent to the error probability of an optimal detector for a binary pulse position modulation (PPM) signal in wireless communications, as a function of the relative delay Δ. Previously, we have adopted the optimal maximum likelihood detector, assuming perfect channel knowledge at the receiver. Then, the average error probability $\overline{P_e}(\Delta)$ over random channels was used to replace $P_e(\Delta)$, leading to an average ZZB [9]. In the following we do not assume the receiver knows the channel, and adopt an LLR test criterion to find the error probability. Employing this in (1) leads to the ZZB.

3. SIGNAL, CHANNEL, AND RECEIVER MODELS

Let the transmitted signal be denoted as

$$s_m(t) = \sqrt{E_{\mathrm{tx}}}p(t-m\Delta), \quad m = 0, 1. \quad (2)$$

where $p(t)$ has unit energy and finite duration T_p. Here, $p(t)$ is arbitrary, with finite extent and normalized energy, and is assumed known at the receiver. This can also be regarded as a binary PPM signal in a communication system, and we proceed to find the probability of bit error. The two hypotheses H_0 and H_1 correspond to $m = 0, 1$, respectively, and Δ is the delay parameter in (1). The convolutive channel is modeled as a tap delay line (TDL) with spacing T_t

$$g(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - lT_t), \quad (3)$$

where L is the total number of taps, and α_l is the gain for the $(l+1)$-th tap modeled as a Gaussian random variable with mean m_l and variance σ_l^2 [9, 12]. We model the α_l's as jointly Gaussian with distribution $\mathcal{N}(\boldsymbol{\mu}_\alpha, \boldsymbol{V})$ where $\boldsymbol{\mu}_\alpha$ is the mean vector and $\boldsymbol{V}$ is the covariance matrix. The channel is assumed to have unit power such that $tr(\boldsymbol{\mu}_\alpha\boldsymbol{\mu}_\alpha^T + \boldsymbol{V}) = 1$, where tr is the trace operator. The received signal is given by

$$y(t) = \sqrt{E_{\mathrm{rx}}}\sum_{l=0}^{L-1} \alpha_l s_m(t - lT_t) + \nu(t), \quad (4)$$

where $\nu(t)$ is AWGN with double sided spectral density $\sigma_\nu^2 = N_0/2$. We assume the received waveform (pulse convolved with the channel) has duration T_o seconds.

The receiver consists of a bank of correlators, with template $p(t) - p(t-\Delta)$ shifted to kT_t for $k = 0, \cdots, L-1$, and integration interval $(kT_t, kT_t + T_o)$. The output of the $(k+1)$th correlator is

$$r_k = \int_{kT_t}^{kT_t+T_o} [p(t-kT_t) - p(t-kT_t-\Delta)]\, y(t)dt. \quad (5)$$

If we denote the pulse autocorrelation by

$$\beta_\tau = \int_0^{T_p} p(t)p(t-\tau)dt = \int_0^\infty p(t)p(t-\tau)dt, \quad (6)$$

then we have

$$r_k = \sqrt{E_{\mathrm{rx}}}\sum_{l=0}^{L-1} \alpha_l \left[\beta_{(k-l)T_t - m\Delta} - \beta_{(k-l)T_t+(1-m)\Delta}\right] + v_k, \quad (7)$$

where v_k is noise given by

$$v_k = \int_{kT_t}^{kT_t+T_o} [p(t-kT_t) - p(t-kT_t-\Delta)]\, \nu(t)dt. \quad (8)$$

We find that the noise samples have the following statistics

$$E\{v_i v_j\} = \sigma_\nu^2 \left[2\beta_{(i-j)T_t} - \beta_{(i-j)T_t-\Delta} - \beta_{(i-j)T_t+\Delta}\right]. \quad (9)$$

Defining $\boldsymbol{r} = [r_0, \cdots, r_{L-1}]^T$, $\boldsymbol{v} = [v_0, \cdots, v_{L-1}]^T$, $\boldsymbol{\alpha} = [\alpha_0, \cdots, \alpha_{L-1}]^T$, and using (7), we obtain the received data vector as

$$\boldsymbol{r} = \sqrt{E_{\mathrm{rx}}}\boldsymbol{R}_m\boldsymbol{\alpha} + \boldsymbol{v}, \quad (10)$$

where

$$\boldsymbol{R}_m = \sum_{k=-(L-1)}^{L-1} (\beta_{kT_t - m\Delta} - \beta_{kT_t+(1-m)\Delta})\boldsymbol{J}_L^k. \quad (11)$$

Here, $\boldsymbol{J}_L$ is an $L \times L$ down-shifting matrix whose first sub-diagonal elements below the main diagonal are all ones, while all others are zeros, $\boldsymbol{J}_L^{-1} \triangleq \boldsymbol{J}_L^T$, $\boldsymbol{J}_L^0 = \boldsymbol{I}$. Denote the covariance matrix of $\boldsymbol{v}$ by $\boldsymbol{\Gamma} = E\{\boldsymbol{v}\boldsymbol{v}^T\} = \sigma_\nu^2 \widetilde{\boldsymbol{\Gamma}}$, where $\widetilde{\boldsymbol{\Gamma}}$ is the normalized noise covariance matrix. We have

$$\widetilde{\boldsymbol{\Gamma}} = \sum_{k=-(L-1)}^{L-1} (2\beta_{kT_t} - \beta_{kT_t-\Delta} - \beta_{kT_t+\Delta})\boldsymbol{J}_L^k, \quad (12)$$

due to (9). These results will be applied to a composite hypothesis test to find the minimum error probability, from which the ZZB follows.

4. THE ZIV-ZAKAI BOUND

The development of the ZZB requires the minimum error probability, that is derived next.

4.1. Minimum Error Probability

The receiver makes a LLR test based on data vector $\boldsymbol{r}$ in order to decide on hypothesis H_0 ($m = 0$) or H_1 ($m = 1$) as follows

$$\Lambda = \ln p(\boldsymbol{r}|H_0) - \ln p(\boldsymbol{r}|H_1) \underset{H_1}{\overset{H_0}{\gtrless}} 0, \quad (13)$$

where $p(\boldsymbol{r}|H_m)$ is the pdf of $\boldsymbol{r}$ conditioned on H_m.

According to (10), $\boldsymbol{r}$ is a function of m and a linear function of $\boldsymbol{\alpha}$ and $\boldsymbol{v}$, which are both Gaussian vectors. So, $\boldsymbol{r}|H_m$ is an $L \times 1$ Gaussian vector with mean $\boldsymbol{\mu}_m = \sqrt{E_{\text{rx}}}\widetilde{\boldsymbol{\mu}}_m$ and covariance $\boldsymbol{C}_m = E_{\text{rx}}\widetilde{\boldsymbol{C}}_m$, where

$$\widetilde{\boldsymbol{\mu}}_m = \boldsymbol{R}_m \boldsymbol{\mu}_\alpha, \quad \widetilde{\boldsymbol{C}}_m = \boldsymbol{R}_m \boldsymbol{V} \boldsymbol{R}_m^T + \frac{1}{\xi_b}\widetilde{\boldsymbol{\Gamma}},$$

and we define $\xi_b = E_{\text{rx}}/\sigma_\nu^2$ as the signal to noise ratio (SNR) at the receiver. Thus, the pdf of $\boldsymbol{r}|H_m$ is

$$p(\boldsymbol{r}|H_m) = \frac{1}{(2\pi)^{\frac{L}{2}}|\boldsymbol{C}_m|^{\frac{1}{2}}} \exp\{-\frac{1}{2}(\boldsymbol{r}-\boldsymbol{\mu}_m)^T \boldsymbol{C}_m^{-1}(\boldsymbol{r}-\boldsymbol{\mu}_m)\}.$$

Our LLR test now becomes

$$\Lambda = \boldsymbol{r}^T \boldsymbol{\Psi} \boldsymbol{r} + \boldsymbol{g}^T \boldsymbol{r} + d \underset{H_1}{\overset{H_0}{\gtrless}} 0, \quad (14)$$

where

$$\boldsymbol{\Psi} = \frac{1}{E_{\text{rx}}}\widetilde{\boldsymbol{\Psi}}, \quad \widetilde{\boldsymbol{\Psi}} = -\frac{1}{2}(\widetilde{\boldsymbol{C}}_0^{-1} - \widetilde{\boldsymbol{C}}_1^{-1}),$$

$$\boldsymbol{g} = \frac{1}{\sqrt{E_{\text{rx}}}}\widetilde{\boldsymbol{g}}, \quad \widetilde{\boldsymbol{g}} = \widetilde{\boldsymbol{C}}_0^{-1}\widetilde{\boldsymbol{\mu}}_0 - \widetilde{\boldsymbol{C}}_1^{-1}\widetilde{\boldsymbol{\mu}}_1,$$

$$d = -\frac{1}{2}\ln\frac{|\widetilde{\boldsymbol{C}}_0|}{|\widetilde{\boldsymbol{C}}_1|} - \frac{1}{2}(\widetilde{\boldsymbol{\mu}}_0^T \widetilde{\boldsymbol{C}}_0^{-1}\widetilde{\boldsymbol{\mu}}_0 - \widetilde{\boldsymbol{\mu}}_1^T \widetilde{\boldsymbol{C}}_1^{-1}\widetilde{\boldsymbol{\mu}}_1).$$

According to (14), an error occurs when $m = 0$ if $\Lambda < 0$. Similarly, an error occurs when $m = 1$ if $\Lambda > 0$. In the following we denote $\boldsymbol{r}|H_m$ by $\boldsymbol{r}_m$ to concisely show its dependence on m. With the dependence on m we have

$$\Lambda_m = \boldsymbol{r}_m^T \boldsymbol{\Psi} \boldsymbol{r}_m + \boldsymbol{g}^T \boldsymbol{r}_m + d. \quad (15)$$

Assume m takes values 0 and 1 with equal probability. The minimum probability of detection error has the form

$$\begin{aligned} P_e(\Delta) &= P\{\Lambda < 0|H_0\}P\{H_0\} + P\{\Lambda > 0|H_1\}P\{H_1\} \\ &= \frac{1}{2}P\{\Lambda_0 < 0\} + \frac{1}{2}P\{\Lambda_1 > 0\}. \end{aligned} \quad (16)$$

From (15) we observe that Λ_m is in a general quadratic form of the Gaussian vector $\boldsymbol{r}_m$, and the resulting pdf of Λ_m may be intractable.

To find the error probability, we use an approach via the moment generating function (MGF) of Λ_m, that can be found as follows [11]

$$\begin{aligned} \Theta_m(s) &= E_{\boldsymbol{r}_m}\{\exp(s\Lambda_m)\} \\ &= E_{\boldsymbol{r}_m}\{\exp[s(\boldsymbol{r}_m^T \boldsymbol{\Psi} \boldsymbol{r}_m + \boldsymbol{g}^T \boldsymbol{r}_m + d)]\} \\ &= |\boldsymbol{A}_m(s)|^{-\frac{1}{2}} \exp\{sk_m + \frac{s^2}{2}\boldsymbol{b}_m^T \boldsymbol{A}_m^{-1}(s)\boldsymbol{b}_m\} \end{aligned} \quad (17)$$

where

$$\begin{aligned} \boldsymbol{A}_m(s) &= \boldsymbol{I} - 2s\boldsymbol{C}_m^{\frac{1}{2}}\boldsymbol{\Psi}\boldsymbol{C}_m^{\frac{1}{2}} \\ &= \boldsymbol{I} + s\widetilde{\boldsymbol{C}}_m^{\frac{1}{2}}(\widetilde{\boldsymbol{C}}_0^{-1} - \widetilde{\boldsymbol{C}}_1^{-1})\widetilde{\boldsymbol{C}}_m^{\frac{1}{2}}, \\ k_m &= d + \boldsymbol{\mu}_m^T \boldsymbol{\Psi} \boldsymbol{\mu}_m + \boldsymbol{g}^T \boldsymbol{\mu}_m \\ &= -\frac{1}{2}\ln\frac{|\widetilde{\boldsymbol{C}}_0|}{|\widetilde{\boldsymbol{C}}_1|} - \frac{1}{2}(\widetilde{\boldsymbol{\mu}}_0^T \widetilde{\boldsymbol{C}}_0^{-1}\widetilde{\boldsymbol{\mu}}_0 - \widetilde{\boldsymbol{\mu}}_1^T \widetilde{\boldsymbol{C}}_1^{-1}\widetilde{\boldsymbol{\mu}}_1) \\ &\quad - \frac{1}{2}\widetilde{\boldsymbol{\mu}}_m^T(\widetilde{\boldsymbol{C}}_0^{-1} - \widetilde{\boldsymbol{C}}_1^{-1})\widetilde{\boldsymbol{\mu}}_m + (\widetilde{\boldsymbol{\mu}}_0^T \widetilde{\boldsymbol{C}}_0^{-1} - \widetilde{\boldsymbol{\mu}}_1^T \widetilde{\boldsymbol{C}}_1^{-1})\widetilde{\boldsymbol{\mu}}_m \\ \boldsymbol{b}_m &= \boldsymbol{C}_m^{\frac{1}{2}}\boldsymbol{g} + 2\boldsymbol{C}_m^{\frac{1}{2}}\boldsymbol{\Psi}\boldsymbol{\mu}_m \\ &= \widetilde{\boldsymbol{C}}_m^{\frac{1}{2}}[(\widetilde{\boldsymbol{C}}_0^{-1}\widetilde{\boldsymbol{\mu}}_0 - \widetilde{\boldsymbol{C}}_1^{-1}\widetilde{\boldsymbol{\mu}}_1) - (\widetilde{\boldsymbol{C}}_0^{-1} - \widetilde{\boldsymbol{C}}_1^{-1})\widetilde{\boldsymbol{\mu}}_m]. \end{aligned}$$

Setting $s = j2\pi f$, the characteristic function of Λ_m, $\Theta_m(f)$, is obtained, whose Fourier transform gives the pdf of Λ_m. Finally the probability $P\{\Lambda_0 < 0\}$ and $P\{\Lambda_1 > 0\}$ can be found from the pdf. Collecting these results, we have

$$P\{\Lambda_0 < 0\} = \int_{-\infty}^{0}\int_{-\infty}^{\infty} \Theta_0(f)e^{-j2\pi f\Lambda_0} df d\Lambda_0, \quad (18)$$

$$P\{\Lambda_1 > 0\} = \int_{0}^{\infty}\int_{-\infty}^{\infty} \Theta_1(f)e^{-j2\pi f\Lambda_1} df d\Lambda_1. \quad (19)$$

Substituting $P\{\Lambda_0 < 0\}$ and $P\{\Lambda_1 > 0\}$ into eq. (16), the minimum error probability $P_e(\Delta)$ follows.

4.2. ZZB

Given the minimum error probability $P_e(\Delta)$ above, we substitute into eq. (1) and integrate with respect to Δ, and so find the desired ZZB on TDE for any system parameters. The bound is applicable to many scenarios, such as line-of-sight (LOS) and non-LOS (NLOS) channels, different power delay profiles (PDPs), tap correlation profiles (TCPs), and pulse shaping, as discussed in [9]. Note that the bound depends on the prior distribution time delay interval T; as $T \to \infty$, it may become loose [3].

4.3. Computational Complexity

The numerical computation of the ZZB requires multidimensional integration. The inner most integration in each of (18) and (19), which is of the characteristic function $\Theta_m(j2\pi f)$ in the frequency domain, produces the pdf of Λ_m. Fast Fourier transform (FFT) algorithms can be employed to approximate the frequency domain integration. Including the multiplication operations, and denoting the FFT length as N_{fft}, then the computational complexity is on the order of $O\{N_{\text{fft}}(4L^2 + 2logN_{\text{fft}})\}$.

For comparison, the corresponding empirical pdf can be obtained by a Monte-Carlo method as follows. First, a two-step procedure is used to generate the samples of the Gaussian random vector $\boldsymbol{r}_m$ in eq. (10) (e.g., see [13]):

Step 1. Simulate the independent samples of a standard Gaussian random vector $\boldsymbol{z}$ with probability distribution $N(\boldsymbol{0}, \boldsymbol{I})$.

Step 2. Compute vector $\boldsymbol{r}_m$ using equation:

$$\boldsymbol{r}_m = \boldsymbol{\mu}_m + \boldsymbol{S}_m \boldsymbol{z}, \;\; m = 0, 1 \tag{20}$$

where $\boldsymbol{S}_m$ is the square-root matrix of the covariance matrix $\boldsymbol{C}_m$ of $\boldsymbol{r}_m$ satisfying $\boldsymbol{S}_m \boldsymbol{S}_m^T = \boldsymbol{C}_m$. $\boldsymbol{\mu}_m$ and $\boldsymbol{C}_m$ are defined in subsection 4.1. Substituting the samples of $\boldsymbol{r}_m$ into eq. (15), we can obtain the samples of Λ_m. Therefore the empirical distribution of Λ_m can be found using these samples. The complexity of this method to obtain the empirical distribution is order of $O(4N_sL^2)$, where L is the number of paths and N_s the number of samples. Thus the complexity of the Monte-Carlo simulation is roughly $\frac{N_s}{N_{fft}}$ times of that of direct numerical integration of the above analytical form.

5. ZZB COMPARISONS

We next study the ZZB behavior for some cases of interest. We assume ultra-wideband (UWB) channels and system parameters similar to those used in [9], and compare the ZZB derived above to the (known channel) average ZZB developed in [9]. In the figures we refer to the ZZB from [9] as the "average ZZB". We plot the root MSE (RMSE) of the time delay estimate as the performance metric. The following parameters are used unless otherwise stated: $T = 30$ns, $T_t = 1$ns, $T_p = 2$ns and a rectangular pulse is transmitted. The channel has $L = 5$ independent taps. Based on [12], an exponential power delay profile is used with decay factor $\lambda = 6$ns. The Ricean-K factor for the first path is $K = 50 \approx 17$dB, and all other paths have a zero Ricean factor (i.e., have zero mean).

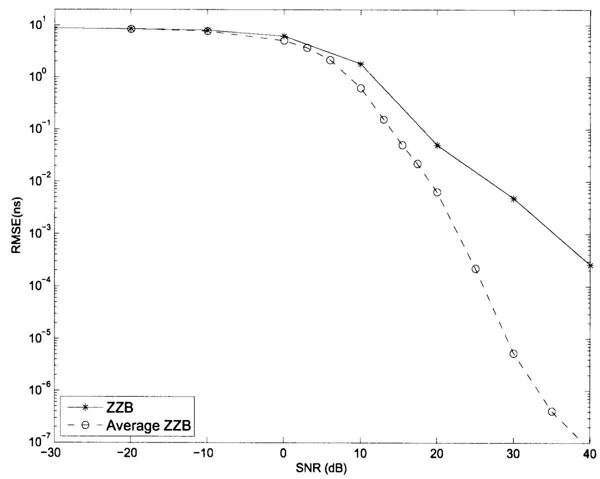

Fig. 1. Comparison of the ZZB developed in this paper with the average ZZB from [9]. $L = 5$, $K = 50$.

Figure 1 compares the ZZB from this paper (solid line with stars) with the average ZZB (dashed line with circles) over a large SNR range. Note that in the low SNR region, the difference of the two bounds is negligible; apparently knowledge of the channel will not significantly help the receiver to recover delay when the receiver is noise dominated. As the SNR increases, the ZZB deviates from the average ZZB and shows worse TDE performance, about one order of magnitude worse at 20dB, and up to three orders worse at 40dB. Thus, in the high SNR region, lack of knowledge of the channel critically impacts time delay estimation performance. However, note that in this scenario the ZZB (RMSE) indicates a relatively low value of 0.050ns at an SNR of 20dB, showing the possibility of discriminating delay to a fraction of the pulse width in a multipath channel.

The detection capability increases as the number of paths decreases, as shown in Fig. 2 for single path channels with varying K factor of $(0, 5, 10, 15, 20)$dB. (Here, a K factor of zero represents a NLOS channel condition.) For example, at 20dB SNR, the ZZB has an RMSE of 0.043ns, or 0.036ns for $K = 15$dB or 20dB. Also, the gaps between the two ZZBs shrink when a comparison between this figure and Fig. 1 is made for a comparable K.

Fig. 3 shows the effect of the uniform prior distribution time delay interval length T, that varies over $(1, 5, 20, 100)$ns.

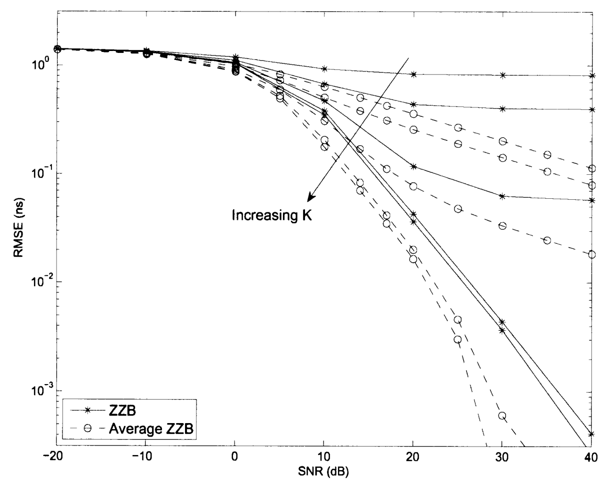

Fig. 2. Effect of K factor on the ZZB and the average ZZB. T_p = 2ns, T = 5ns, L = 1. Curves are parameterized by Ricean-K factor in the order of $(0, 5, 10, 15, 20)$dB.

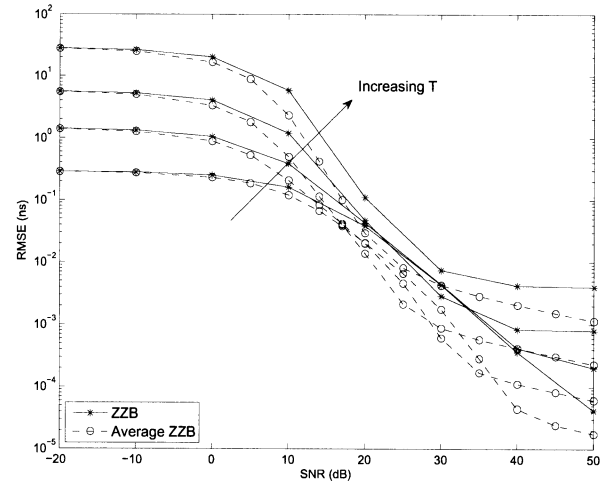

Fig. 3. Effect of time delay interval length T on the ZZB and the average ZZB. L = 1, K = 15dB. Curves are parameterized by T in the order of $(1, 5, 20, 100)$ns.

The departure gap of the ZZB from the average ZZB, and the error level, tend to increase at high SNRs as T increases. It is therefore important to carefully choose T when modeling TDE problems.

6. CONCLUSIONS

In this paper we have developed a Ziv-Zakai bound on TDE for convolutive random channels unknown to the receiver. Compared with the average ZZB that assumes perfect knowledge of the channel realization at the receiver, the ZZB developed in this paper predicts a much higher TDE error. Differences between the two bounds are pronounced at high SNRs, but decrease when the number of channel paths decreases or when the first path Ricean-K factor increases (that is, in the presence of a strong LOS component). This comparison shows when channel estimation will have a more significant effect on TDE.

Future topics of interest include comparisons with TDE estimator's MSE performance, as well as analytical study of the low and high SNR asymptotic ZZB behavior.

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