Low-Complexity Hyperbolic Source Localization With A Linear Sensor Array

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***Abstract*—We develop linear equations for low-complexity source localization, based on time difference of arrival (TDOA) measurements at a linear sensor array. The derivation comes from hyperbolic geometry in Cartesian coordinates, and the algorithm is valid for (possibly wideband) sources in the near or far field. An error analysis is used to predict bias and mean-square error, as well as provide optimal weighting for a low-complexity noniterative weighted least squares solution. A connection is made with an algorithm for arbitrary array geometry. Simulation results show that the proposed algorithm achieves maximum-likelihood performance and the Cramér–Rao bound at medium to low TDOA measurement noise level.**

***Index Terms*—Hyperbolic, linear array, source localization.**

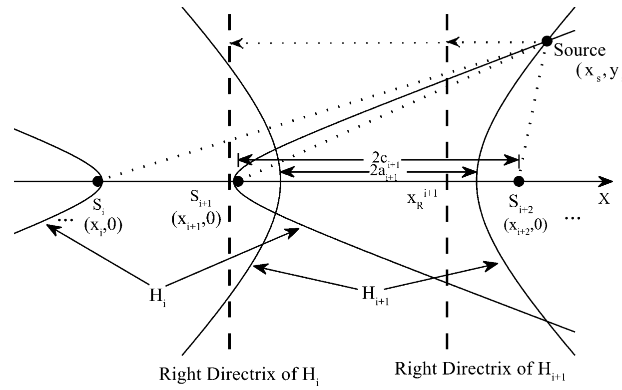

Fig. 1. An M-sensor linear array. The signal source is at (x_s, y_s).

I. Introduction

Time difference of arrival (TDOA) estimates from a sensor array can be used to localize a source, e.g., see Lee [1]. This TDOA-based localization technique accommodates wideband signals, spans both near and far field sources, and requires relatively little *a priori* information about the source. The problem arises in acoustics and other applications, and it requires sufficient time-bandwidth product, signal-to-noise ratio, and spatial coherence [2]. The linearization solution based on Taylor-series expansion by Foy [3] involves iterative processing, typically incurs high computational complexity, and convergence requires a tolerable initial estimate of the position. Generalized cross-correlator processing techniques have been proposed and analyzed by Carter [4], and methods based on range-differences have also been developed [5].

In practice, low-complexity solutions may be highly desirable, to conserve energy and reduce processing delay. One way to achieve this is to set up and solve a set of linear equations, thereby yielding fixed complexity with classic least-square solutions.

Linear hyperbolic positioning methods based on TDOAs for an arbitrary receiving array were proposed independently in [5] and [6]. They are not iterative and do not suffer from initialization issues. Some closed-form linear algorithms were also proposed for passive acoustic source localization applications [7], [8]. A noniterative closed-form algorithm for 2-D geolocation was presented by Chan and Ho [9]; this work also established that the two earlier algorithms ([5] and [6]) were equivalent and suboptimal. The main idea is a two-stage weighted least-squares (WLS) technique using a refining process at the second stage by employing the nonlinear relation of the coordinates. This WLS algorithm achieves the Cramér–Rao bound (CRB) for small TDOA measurement errors.

In this paper, we further analyze and develop a simple closed-form algorithm for a linear sensor array [10], by utilizing geometric properties of hyperbolas. A WLS estimator is proposed whose performance and improved weighting matrices are derived based on a second-order perturbation technique, and we show a connection with the localization algorithm of Friedlander [6]. Simulations confirm the analysis and show that the algorithm achieves the CRB when the TDOA error is sufficiently small.

II. Hyperbolic Linear Localization Algorithm

Consider a size M linear sensor array, from S_1 to S_M, placed on the x-axis as shown in Fig. 1. The (x,y) coordinates of S_i are $\boldsymbol{p}_i = [x_i, y_i]^T$, where $y_i = 0$. The signal source is at $\boldsymbol{p}_s = [x_s, y_s]^T$. The hyperbola H_i centered in the middle of two sensors S_i and S_{i+1} is determined by the TDOA τ_i between them. Thus, the range difference $d_{i,i+1}$ from the TDOA τ_i is given by

$$d_{i,i+1} = v_c \tau_i = r_i - r_{i+1},\ i = 1, 2, \ldots, M-1 \quad (1)$$

where $r_i \triangleq \|\boldsymbol{p}_i - \boldsymbol{p}_s\|$ is the distance from the signal source to the ith sensor. In the following, to simplify the notation, we replace $d_{i,i+1}$ with d_i.

The standard geometry definition of hyperbola is: the difference of the distance from any point on the hyperbola to the two foci is $2a$. If the distance between the two foci is $2c$ and we adopt a Cartesian coordinate system, we have

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2a$$

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or an equivalent form $x^2/a^2 - y^2/b^2 = 1$, where $b = \sqrt{c^2 - a^2}$.

As we have shown [10], a closed-form solution for x_s can be derived directly from the alternative definition of a hyperbola: the locus of points whose distance from the focus is proportional to the horizontal distance from a vertical line known as the conic section directrix, where the ratio is the eccentricity. The signal source lies on a series of hyperbolas whose parameters are determined by d_i as well as the distance between the two respective receiving sensors, as shown in Fig. 1. The parameters of hyperbola H_i can be obtained as

$$a_i = \frac{|d_i|}{2},\ c_i = \frac{x_{i+1} - x_i}{2},\ e_i = \frac{c_i}{a_i} \tag{2}$$

$$x_L^i = \frac{x_i + x_{i+1}}{2} - \frac{a_i^2}{c_i},\ x_R^i = \frac{x_i + x_{i+1}}{2} + \frac{a_i^2}{c_i} \tag{3}$$

where e_i is the eccentricity of H_i, and x_L^i and x_R^i are the x-coordinates of intersections of the left and right directrices of H_i with the x-axis, respectively.

Since sensor $S_{i+1}(i = 1, 2, \ldots, M-2)$ is both the right focus of hyperbola H_i and the left focus of hyperbola H_{i+1}, the distance from the source to this sensor can be equivalently expressed in terms of the directrices and eccentricities of both H_i and H_{i+1} as

$$e_i|x_s - x_R^i| = e_{i+1}|x_s - x_L^{i+1}|. \tag{4}$$

Note that $x_R^i < x_L^{i+1}$ always holds. Let ν_i be an indicator of relative location of source and sensor, equal to -1 when $x_R^i < x_s < x_L^{i+1}$, and 1 otherwise. Now, we can write

$$e_i(x_s - x_R^i) = \nu_i(e_{i+1}x_s - e_{i+1}x_L^{i+1}). \tag{5}$$

Substituting (2) and (3), then (5) becomes

$$\left[\frac{x_{i+1} - x_i}{|d_i|} - \frac{\nu_i(x_{i+2} - x_{i+1})}{|d_{i+1}|}\right]x_s = \frac{x_{i+1}^2 - x_i^2}{2|d_i|} - \frac{\nu_i(x_{i+2}^2 - x_{i+1}^2)}{2|d_{i+1}|} + \frac{\nu_i|d_{i+1}| + |d_i|}{2}. \tag{6}$$

Collecting coefficients of x_s corresponding to $i = 1, \ldots, M-2$ in a column vector $\boldsymbol{e}$, and the right-hand side in a column vector $\boldsymbol{g}$, the localization model can be written as

$$\boldsymbol{e}x_s = \boldsymbol{g}. \tag{7}$$

Noting that all quantities except x_s are known, then x_s can be estimated via a weighted least-squares criterion given by

$$\begin{aligned}\hat{x}_s &= \arg\min_{x_s}(\boldsymbol{g} - x_s\boldsymbol{e})^T\boldsymbol{W}_x(\boldsymbol{g} - x_s\boldsymbol{e})\\ &= (\boldsymbol{e}^T\boldsymbol{W}_x\boldsymbol{e})^{-1}\boldsymbol{e}^T\boldsymbol{W}_x\boldsymbol{g}.\end{aligned} \tag{8}$$

We choose the weighting matrix as $\boldsymbol{W}_x = \boldsymbol{\Phi}_x^{-1}$, where $\boldsymbol{\Phi}_x$ is the covariance matrix of the error vector

$$\boldsymbol{\Phi}_x = E\{\boldsymbol{\varPhi}_x\boldsymbol{\varPhi}_x^T\},\ \boldsymbol{\varPhi}_x = \boldsymbol{\delta g} - x_s\boldsymbol{\delta e}. \tag{9}$$

An expression for $\boldsymbol{\Phi}_x$ is found in Section IV below.

Once x_s is estimated, y_s can be found from a set of equations describing $M-1$ consecutive hyperbolas

$$\frac{y_s^2}{b_i^2} = \frac{(x_s - \frac{x_i + x_{i+1}}{2})^2}{a_i^2} - 1 \tag{10}$$

where $b_i^2 = c_i^2 - a_i^2$. Stacking elements for $i = 1, \ldots, M-1$ into vectors, we obtain

$$\boldsymbol{f}y_s^2 = \boldsymbol{h}. \tag{11}$$

This immediately suggests a WLS estimate for y_s^2

$$\begin{aligned}\hat{y_s^2} &= \arg\min_{y_s^2}(\boldsymbol{h} - y_s^2\boldsymbol{f})^T\boldsymbol{W}_y(\boldsymbol{h} - y_s^2\boldsymbol{f})\\ &= (\boldsymbol{f}^T\boldsymbol{W}_y\boldsymbol{f})^{-1}\boldsymbol{f}^T\boldsymbol{W}_y\boldsymbol{h}\end{aligned} \tag{12}$$

where $\boldsymbol{W}_y$ is the weighting matrix, and an estimate of y_s follows. $\boldsymbol{W}_y$ can be chosen as $\boldsymbol{\Phi}_y^{-1}$, where

$$\boldsymbol{\Phi}_y = E\{\boldsymbol{\varPhi}_y\boldsymbol{\varPhi}_y^T\},\ \boldsymbol{\varPhi}_y = \boldsymbol{\delta h} - y_s^2\boldsymbol{\delta f}. \tag{13}$$

For best performance, the weighting matrices $\boldsymbol{W}_x$ and $\boldsymbol{W}_y$ depend on the true source location, and they can be found either by an iterative process based on the previous estimated location [6] or from the signal and noise power spectra [9]. In our simulations, we will use the theoretical error analysis results detailed in Section IV to build the optimal weighting matrices.

The inherent reflection ambiguity with a linear array means the polarity of y_s, and the value of ν_i, depend on the source location relative to the array. In practice, some prior measurement or other information can be exploited. For example, to determine ν_i, compare r_i, r_{i+1}, and r_{i+2}. If $r_{i+1} < r_i$ and $r_{i+1} < r_{i+2}$, then $\nu_i = -1$; otherwise, $\nu_i = 1$. In the following development, we restrict our attention to the case where the source is located above the sensor array, leading to $\nu_i = 1$ for all $i = 1, \ldots, M-2$ and $d_i > 0$ for all $i = 1, \ldots, M-1$. We also assume $y_s > 0$ to further resolve the ambiguity in estimating the source y-coordinate. This discussion is easily generalized to other scenarios.

III. Connection With an Algorithm for Arbitrary Arrays

Next we show a link with a solution proposed from a different perspective [6] that is first briefly reviewed. Consider an arbitrary sensor array. Denote the sensor coordinates as $\boldsymbol{p}_i = [x_i, y_i]^T$, where $y_i = 0$ does not always hold. The distance from the origin to S_i is $R_i \triangleq \|\boldsymbol{p}_i\|$. To be consistent with notation in [6], in this section, we use $d_{i,j}$ for range-difference and $\tau_{i,j}$ for TDOA.

In [6], a nuisance parameter r_j is introduced, that is the distance from the source to an arbitrary reference sensor S_j in the array. The algorithm derivation starts from the definition of r_i and range difference $d_{i,j}$. After some algebraic manipulation, collecting $M-1$ equations for i into a matrix form, and pre-multiplying by a matrix $\boldsymbol{M}_j$, it is shown that [6]

$$\boldsymbol{M}_j\boldsymbol{S}_j\boldsymbol{p}_s = \boldsymbol{M}_j\boldsymbol{\mu}_j \tag{14}$$

where $\boldsymbol{M}_j$ is a null-space matrix of $\boldsymbol{\rho}_j(\boldsymbol{M}_j\boldsymbol{\rho}_j = 0)$, and the ith rows of $\boldsymbol{S}_j$, $\boldsymbol{\mu}_j$, and $\boldsymbol{\rho}_j$ are $(\boldsymbol{p}_i - \boldsymbol{p}_j)^T$, $(R_i^2 - R_j^2 - d_{i,j}^2)/2$, and $d_{i,j}$, respectively. A closed-form solution is obtained as

$$\boldsymbol{p}_s = (\boldsymbol{S}_j^T\boldsymbol{M}_j^T\boldsymbol{M}_j\boldsymbol{S}_j)^{-1}\boldsymbol{\mu}_j^T\boldsymbol{M}_j^T\boldsymbol{M}_j\boldsymbol{\mu}_j \tag{15}$$

as long as matrix $\boldsymbol{M}_j\boldsymbol{S}_j$ has full column rank, which is highly probable for an arbitrary array of sufficient size. Note, however, that this algorithm excludes a linear array geometry because in this case, $\boldsymbol{M}_j\boldsymbol{S}_j$ is singular.

In particular, for a linear array as in Fig. 1, the location of S_i becomes $\boldsymbol{p}_i = [x_i, 0]^T$. Thus, matrix $\boldsymbol{S}_j$ has an all-zero column and loses column rank. Now, from (14), only x_s can be determined, and y_s must be estimated by another method, such as via (12). For simplicity, suppose we choose the sensor S_1 as the reference sensor. The hyperbola H_i with two foci S_1 and S_i is determined by the range difference $d_{i,1}$. Substituting expressions for $\boldsymbol{M}_j$, $\boldsymbol{S}_j$, and $\boldsymbol{\mu}_j$ into (14), we obtain $M-1$ equations, but with only $M-2$ independent equations. Excluding the last equation, the remaining $M-2$ equations can be arranged as

$$\bar{\boldsymbol{e}} x_s = \bar{\boldsymbol{g}} \tag{16}$$

where the ith element of $\bar{\boldsymbol{e}}$ is $(x_{i+1}-x_1)/d_{1,i+1} - (x_{i+2}-x_1)/d_{1,i+2}$, and the ith element of $\bar{\boldsymbol{g}}$ is $(x_{i+1}^2 - x_1^2)/2d_{1,i+1} - (x_{i+2}^2 - x_1^2)/2d_{1,i+2} + (d_{1,i+2} - d_{1,i+1})/2$, for $i = 1, \ldots, M-2$.

Referring back to Section II, if we also use S_1 as the reference sensor, then the range difference becomes $d_{1,i}$, $(i \neq 1)$, for sensors S_1 and S_i. The hyperbola H_i has the following parameters:

$$\bar{a}_i = \frac{|d_{1,i}|}{2},\ \bar{c}_i = \frac{x_i - x_1}{2},\ \bar{e}_i = \frac{\bar{c}_i}{\bar{a}_i} = \frac{x_i - x_1}{|d_{1,i}|} \tag{17}$$

$$\bar{x}_L^i = \frac{x_1 + x_i}{2} - \frac{\bar{a}_i^2}{\bar{c}_i} = \frac{x_1 + x_i}{2} - \frac{d_{1,i}^2}{2(x_i - x_1)}. \tag{18}$$

Now the distance r_1 from the source to the first sensor S_1 can be written in $M-1$ equations as

$$\begin{aligned} r_1 &= (x_s - \bar{x}_L^i)\bar{e}_i \\ &= \left[x_s - \frac{x_1 + x_i}{2} + \frac{d_{1,i}^2}{2(x_i - x_1)}\right]\frac{x_i - x_1}{d_{1,i}} \\ &= \frac{x_i - x_1}{d_{1,i}} x_s - \frac{x_i^2 - x_1^2}{2d_{1,i}} + \frac{d_{1,i}}{2}. \end{aligned} \tag{19}$$

Subtracting two consecutive equations for $i = 2, \ldots, M$, we obtain $M-2$, which can be written in a matrix form identical to (16). Therefore, our proposed solution for x_s is mathematically equivalent to a modified version of the algorithm in [6] when the sensor array is linear and the reference sensor is fixed.

Our derivation provides an intuitive view and a geometric interpretation, and we have developed an estimator for y_s. In the next section, we develop bias and mean-square error expressions for our algorithm.

IV. Error Analysis

First, we find the mean-square error (MSE) and bias of $\hat{x}_s$, following a procedure similar to [10] but with weighting matrices incorporated. The estimation error may be caused by inaccurate distance difference (or equivalently TDOA) measurements or possibly sensor location error [11]. Here we consider the impact of measurement errors $\delta\boldsymbol{d} = [\delta d_1, \ldots, \delta d_{M-1}]^T$. For simplicity, we assume the errors are zero mean and independent with diagonal correlation matrix $\boldsymbol{C}_d = \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_{M-1}^2\}$. Analyses can be easily generalized to incorporate other cases. Denoting a squared error vector as $\boldsymbol{\epsilon} = [(\delta d_1)^2, \ldots, (\delta d_{M-1})^2]^T$ and applying a second-order perturbation technique, errors in $\boldsymbol{e}$ and $\boldsymbol{g}$ from (8) can be found to take the following form:

$$\delta\boldsymbol{b} \approx \boldsymbol{P}\delta\boldsymbol{d} + \boldsymbol{R}\boldsymbol{\epsilon},\ \delta\boldsymbol{g} \approx \boldsymbol{Q}\delta\boldsymbol{d} + \boldsymbol{S}\boldsymbol{\epsilon} \tag{20}$$

which are functions of deterministic matrices that depend on the error-free measurements and sensor locations and whose expressions are provided in [10]. We find the covariance matrix $\boldsymbol{\Phi}_x$, defined in (9), to be

$$\boldsymbol{\Phi}_x \approx (\boldsymbol{Q} - x_s\boldsymbol{P})\boldsymbol{C}_d(\boldsymbol{Q} - x_s\boldsymbol{P})^T \tag{21}$$

whose inverse we use for weighting matrix $\boldsymbol{W}_x$. Replacing $\boldsymbol{e}$ by $\boldsymbol{e}+\delta\boldsymbol{e}$ and $\boldsymbol{g}$ by $\boldsymbol{g}+\delta\boldsymbol{g}$ in (8), and substituting (20), the estimation error for x_s up to the second order of $\delta\boldsymbol{d}$ can be easily found by Taylor expansion as

$$\delta x_s \approx \boldsymbol{u}^T\delta\boldsymbol{d} + \delta\boldsymbol{d}^T\boldsymbol{V}\delta\boldsymbol{d} + \boldsymbol{w}^T\boldsymbol{\epsilon} \tag{22}$$

where $\boldsymbol{u}$, $\boldsymbol{V}$, and $\boldsymbol{w}$ are functions of error-free measurements, sensor locations, and weighting matrix $\boldsymbol{W}_x$, similar to those defined in [10] but with the appropriate insertion of $\boldsymbol{W}_x$. Now, the bias β_x and the MSE γ_x^2 of x_s are found to be

$$\beta_x = E\{\delta x_s\} \approx tr\{\boldsymbol{V}\boldsymbol{C}_d\} + \boldsymbol{w}^T\boldsymbol{C}_d\boldsymbol{1} \tag{23}$$

$$\gamma_x^2 = E\{(\delta x_s)^2\} = E[(\hat{x}_s - x_s)^2] \approx \boldsymbol{u}^T\boldsymbol{C}_d\boldsymbol{u} \tag{24}$$

where $\boldsymbol{1} = [1, \ldots, 1]^T$.

To obtain the estimation error for y_s, we first consider y_s^2. Notice that (12) has a form similar to (8). Similar to our previous development, we find $\delta\boldsymbol{f}$ and $\delta\boldsymbol{h}$, and then $\delta(y_s^2)$ is given by

$$\delta(y_s^2) \approx \boldsymbol{\eta}^T\delta\boldsymbol{d} + \delta\boldsymbol{d}^T\boldsymbol{T}\delta\boldsymbol{d} + \boldsymbol{\lambda}^T\boldsymbol{\epsilon} \tag{25}$$

where deterministic quantities $\boldsymbol{\eta}$, $\boldsymbol{T}$, and $\boldsymbol{\lambda}$ are similar to those obtained in [10], but now incorporating $\boldsymbol{W}_y$. Finally, using $\delta\boldsymbol{f}$ and $\delta\boldsymbol{h}$, the covariance matrix $\boldsymbol{\Phi}_y$ defined in (13) is found, and its inverse used as $\boldsymbol{W}_y$.

Since $\delta(y_s^2) \approx 2y_s\delta y_s$, the bias β_y and MSE γ_y in estimating y_s can be found in terms of $\delta(y_s^2)$ as follows:

$$\beta_y = E\{\delta y_s\} \approx \frac{E\{\delta(y_s^2)\}}{2y_s} \approx \frac{tr\{\boldsymbol{T}\boldsymbol{C}_d\} + \boldsymbol{\lambda}^T\boldsymbol{C}_d\boldsymbol{1}}{2y_s} \tag{26}$$

$$\gamma_y = E\{(\delta y_s)^2\} \approx E\left\{\left(\frac{\delta(y_s^2)}{2y_s}\right)^2\right\} \approx \frac{\boldsymbol{\eta}^T\boldsymbol{C}_d\boldsymbol{\eta}}{4y_s^2}. \tag{27}$$

Given (23), (24), (26), and (27), the MSE and bias of the total localization error can be obtained as (e.g., see [11])

$$\gamma^2 = \frac{(\gamma_x^2 + \gamma_y^2)}{r_0^2},\ \beta = \frac{\sqrt{\beta_x^2 + \beta_y^2}}{r_0} \tag{28}$$

and the total variance is given by $\gamma^2 - \beta^2$.

V. Simulation Results

Consider a ten-element array with sensors located at $(x_i, y_i) = (i-1, 0)$, $i = 1, \ldots, 10$. The source is located at $(x_0, y_0) = (17,22)$. The additive noise in the distance difference measurements is assumed zero mean, independent Gaussian distributed, and the noise variance for all sensor pairs is σ^2. Fig. 2 compares the average normalized MSEs and bias between the LS (identity weighting matrix) and WLS

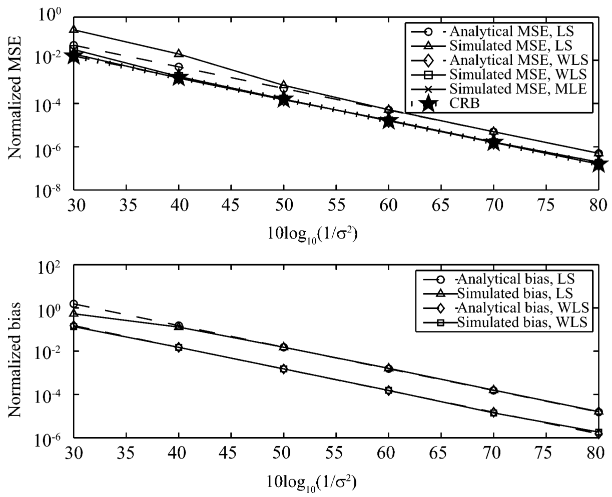

Fig. 2. Performance comparisons for LS, WLS, MLE, and CRB.

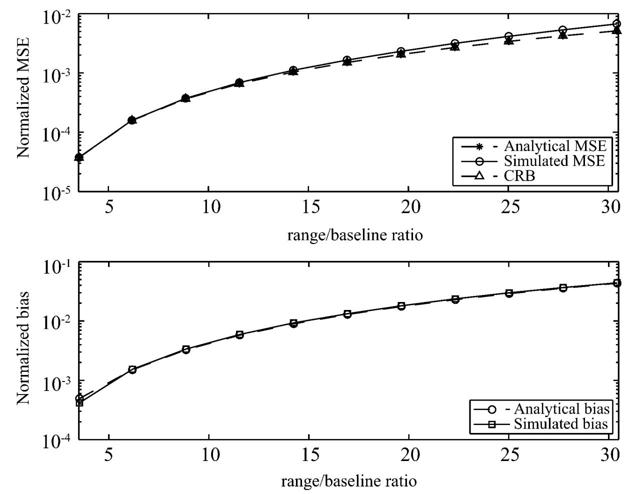

Fig. 4. Performance of the WLS algorithm for varying range/baseline ratios.

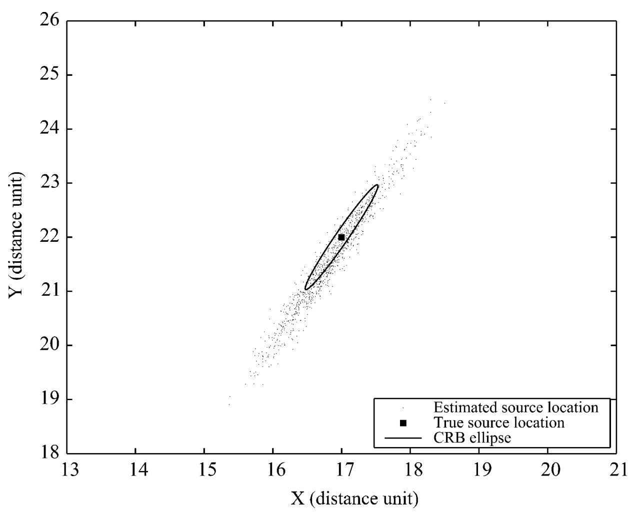

Fig. 3. WLS estimated source locations over 1000 noise realizations.

algorithms versus $10\log_{10}(1/\sigma^2)$ from 500 000 independent realizations and also shows the approximate MLE performance [11] and CRB [9]. The WLS performance coincides with the MLE at about 40 dB, with much lower complexity. WLS shows a 5-dB gain compared to LS for a large range of σ^2. The squared bias of the WLS algorithm is insignificant compared with the respective MSE from 40 dB, such that the MSEs of the WLS and MLE coincide with the CRB at low measurement variance. Fig. 3 presents the CRB ellipse [12] and a scatter plot of the WLS location estimates from 1000 independent realizations at $10\log_{10}(1/\sigma^2) = 40$ dB. It is visually evident that the WLS is asymptotically unbiased. Note that the error in angle is much smaller than that in range, which results from the source/array geometry. Fig. 4 presents the performance of the WLS algorithm versus the ratio of the source distance to the half size of the sensor array ($r_0/L/2$). The array length is 9 distance units and $10\log_{10}(1/\sigma^2) = 50$ dB. The MSE achieves 8×10^{-5} and 5×10^{-3} with a range/baseline ratio of 5 and 30, respectively. As the source moves to the far field, the range estimate becomes less and less accurate, while the angle error remains small.

VI. Conclusion

A low-complexity noniterative WLS source localization algorithm was derived for a linear array, based on geometric properties of a set of hyperbolas. Error performance was conducted using a perturbation technique, and WLS performance approaches the CRB for medium to low measurement noise levels, which verified the algorithm is asymptotically optimal. The algorithm provides a simple method for localization for both near and far-field sources.

References

[1] H. Lee, “A novel procedure for assessing the accuracy of hyperbolic multilateration systems,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-11, pp. 2–15, Jan. 1975.

[2] R. J. Kozick and B. M. Sadler, “Source localization with distributed sensor arrays and partial spatial coherence,” *IEEE Trans. Signal Process.*, vol. 52, no. 3, pp. 601–616, Mar. 2004.

[3] W. H. Foy, “Position-location solutions by Taylor-series estimation,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-12, pp. 187–194, Mar. 1976.

[4] G. Carter, “Time delay estimation for passive sonar signal processing,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-29, pp. 463–470, Jun. 1981.

[5] J. O. Smith and J. S. Abel, “Closed-form least-squares source location estimation from range-difference measurements,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-35, pp. 1661–1669, Dec. 1987.

[6] B. Friedlander, “A passive localization algorithm and its accuracy analysis,” *IEEE J. Ocean. Eng.*, vol. 12, no. 1, pp. 234–245, Jan. 1987.

[7] Y. Huang, J. Benesty, G. W. Elko, and R. M. Mersereati, “Real-time passive source localization: a practical linear-correction least-squares approach,” *IEEE Trans. Speech Audio Process.*, vol. 9, pp. 943–956, Nov. 2001.

[8] M. S. Brandstein, J. E. Adcock, and H. F. Silverman, “A closed-form location estimator for use with room environment microphone arrays,” *IEEE Trans. Speech Audio Process.*, vol. 5, no. 1, pp. 45–50, Jan. 1997.

[9] Y. T. Chan and K. C. Ho, “A simple and efficient estimator for hyperbolic location,” *IEEE Trans. Signal Process.*, vol. 42, no. 8, pp. 1905–1915, Aug. 1994.

[10] Z. Xu, N. Liu, and B. M. Sadler, “A simple closed-form linear source localization algorithm,” in *Proc. MILCOM*, Oct. 2007.

[11] R. J. Barton and D. Rao, “Performance capabilities of long-range UWB-IR TDOA localization systems,” in *EURASIP J. Adv. Signal Process.*, 2008, vol. 2008, Article ID 236791.

[12] B. M. Sadler, R. J. Kozick, and L. Tong, “Multi-modal sensor localization using a mobile access point,” in *Proc. ICASSP*, Mar. 2005.