The Research on the Detection Algorithms for Layered Space-time Codes[1]

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Abstract--**Layered space-time (LST) architecture is effective in achieving high data rate in wireless communication. Zero-Forcing (ZF) and Minimum Mean-Squared Error (MMSE) algorithms are two common detection algorithms for LST codes. However, both ZF and MMSE require the number of receive antennas no less than that of transmit antennas, which greatly limits the application of LST codes in mobile communication. In this paper we introduce Maximum Likelihood (ML) detection algorithm for LST codes, which has no limit on the number of antenna, and compare the performance of the three detection methods for layered space-time codes over flat Rayleigh fading channel. Their applicability is then pointed out respectively.**

Index Terms-- **Layered space-time codes；Zero-Forcing algorithm; Minimum Mean-Squared Error algorithm; Maximum Likelihood detection**

I. INTRODUCTION

Layered space-time (LST) architecture, originally proposed by *Foschini* in [1], is the first kind of space-time codes model. It was widely noticed for its great potential performance in improving spectral efficiency and has shown a bright application future as a solution in MIMO (multi-input multi-output) system to provide high-speed wireless packet data service.

All of the three kinds of space-time codes, namely, space-time trellis codes, space-time block codes and layered space-time codes, assume the availability of accurate channel estimates, which is required for decoding. In comparison with the other two kinds of space-time codes, LST codes can achieve very high spectral efficiency at the cost of partial diversity gain. It can be proved that [2], in a system with N transmit antennas and M receive antennas, the channel capacity which LST codes can achieve will increase linearly with min(M,N) provided the channel is estimated accurately and the fading coefficients are independent. So far the LST codes are the only coding scheme that can make spectral efficiency increase linearly with the number of antennas. This fact has brought the LST codes great advantages in the high-speed wireless communication.

Since the LST codes cannot obtain the maximum diversity gain, the detection algorithm on the receiving side is very significant to the entire system performance. In [3] the performance of the two coding schemes of the LST codes, HLST (Horizontal Layered Space-Time) and DLST (Diagonal Layered Space-Time), is compared and the two detection algorithms, ZF and MMSE

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algorithms, for them are discussed. In this paper we present a comprehensive study of the three detection algorithms for the LST codes, including the performance over flat fading and Rayleigh fading channels, the applicability and the application environment of each algorithm. The paper is organized as follows. In Section Ⅱ we present the system model and establish notations. In section Ⅲ, we describe ZF and MMSE and introduce the ML algorithm for LST, while emphasis is put on the comparison of the applicability of the three algorithms. In section Ⅳ the frame error rate(FER) and bit error rate(BER) performances as a function of signal-to-noise ratio (SNR) are simulated under the quasi-static fading environment according to 3GPP channel model and the simulation results are analysed. In section Ⅴ we cope with the application environment for the three algorithms respectively and draw an important conclusion.

II. SYSTEM MODEL

Fig.1 shows a layered space-time coding system model with N transmit and M receive antennas. In fact the process of LST coding corresponds to realizing a mapping. LST codes, according to different mapping modes, can be classified into VLST (Vertical Layered Space-Time), HLST and DLST codes. Because they have the common basic coding principles, we only consider VLST here.

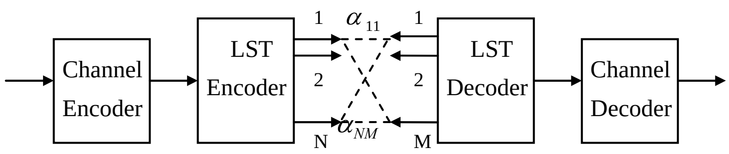

Fig.1. LST codes system model

Over the flat fading channel, the received signal on the j-th antenna at time t, denoted by r_t^j, $j=1,2,\ldots,M$, is given by

$$r_j^t = \sqrt{E_s}\sum_{i=1}^{N}\alpha_{i,j}c_j^t + \eta_j^t \tag{1}$$

where the coefficient $\alpha_{i,j}$ is the path gain which is modeled as the sample of independent zero mean complex Gaussian random variable with variance 0.5 per dimension. The noise samples η^t are modeled as independent samples of a zero mean complex Gaussian random variable with variance $\sigma^2 = N_0/2$ per dimension. Suppose that the elements of the signal constellation are constructed by a factor of $\sqrt{E_s}$ chosen so that the average energy of the constellation is 1. For simplicity, the subscript t is dropped.

Equation (1) can be written in the matrix form

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{c} + \boldsymbol{\eta} \tag{2}$$

where $\boldsymbol{r} = (r_1, r_2, \ldots r_M)^T$ denotes the receive signal vector of M dimensions, $\boldsymbol{c} = (c_1, c_2, \ldots c_N)^T$, the transmit signal vector of N dimensions, $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots \eta_M)^T$, the additive noise vector of M dimensions, and $\boldsymbol{H}$ the $M \times N$ channel matrix.

The receiver can recover the signals only through the information supplied by the channel matrix $\boldsymbol{H}$ because the process of LST decoding is just simple serial-to-parallel conversion and the transmit signal on each antenna is independent. The three detection algorithms deal with the channel matrix in different ways.

III. THREE DETECTION ALGORITHMS FOR LST CODES

A. ZF and MMSE Algorithms

Assume that the ordered set $S = \{k_1, k_2, ..., k_N\}$ denotes a permutation of the integers $1,2,\cdots,N$ specifying the order where the components of the transmitted symbol vector $\boldsymbol{c}$ are extracted. Then the full ZF or MMSE detection algorithm can be described as a recursive procedure [1].

In order to remove the fading corruption, both ZF and MMSE algorithms choose weight vectors for each received sub-sequence to satisfy some performance-related criterion such as ZF or MMSE criterion. On the receiving side the weight vectors for all the N transmit signals can form a matrix

$$\boldsymbol{W} = (w_1, w_2, ...w_N)^T \tag{3}$$

For either ZF or MMSE algorithm, the essential problem lies in how to choose weight vectors for each receive antenna according to the received signals and the channel matrix. The unique difference of these two algorithms is the rule of choosing weight vectors.

In ZF algorithm, zero-forcing criterion is used to choose the N weight vectors of M dimensions $\boldsymbol{w}_{k_i}, i = 1,2,...N,$ which satisfy

$$\boldsymbol{w}_{k_i}{}^T (\boldsymbol{H})_{k_j} = \delta_{ij} = \begin{cases} 0, j \neq i \\ 1, j = i \end{cases} \tag{4}$$

Thus w_{k_i} is orthogonal to the subspace spanned by the contributions to $\boldsymbol{r}$ due to those symbols not yet estimated and cancelled. Obviously, the vector satisfying (4) is just the k_i-th row of $\boldsymbol{H}^{+}_{\bar{k}_{i-1}}$ where the notation $\boldsymbol{H}^{-}_{k_{i-1}}$ represents the matrix obtained by zeroing columns $k_1, k_2, \ldots, k_i$ of $\boldsymbol{H}$ and "+" denotes the Moore-Penrose pseudo inverse.

In MMSE algorithm the MMSE criterion is used to choose the weight vectors. That is, the weight vector matrix $\boldsymbol{W}$ minimizes the mean-squared error $E\{\|\boldsymbol{c} - \boldsymbol{W}\boldsymbol{r}\|^2\}$, which results in

$$\boldsymbol{W}_{MMSE} = \boldsymbol{H}^* \left(\boldsymbol{H}\boldsymbol{H}^* + \sigma^2 \boldsymbol{I}\right)^+ \tag{5}$$

where * denotes conjugate transition.

B. The ML Detection Algorithm

ML decoding is optimum in terms of achieving the lowest error probability. In [5] *Stefanov* proposed an ML decoding algorithm for Turbo-coded system with antenna diversity over block fading channels. That system has similar architecture with the Turbo-coded VLST system. Therefore, ML decoding can also be used in VLST system.

Assume the size of the constellation is 2^b, and $\{c_i\}_{i=1}^{2^b}$ denotes the set of constellation points. The received signal by antenna j at time t is given by

$$r_j = \alpha_{1,j} c_1 + \alpha_{2,j} c_2 + \cdots + \alpha_{N,j} c_N + \eta_j$$

In each transmit interval the received signal is Nb bits information on the j-th receive antenna if there are N transmit antennas. Let us denote the Nb bits that construct $c_1, c_2, \cdots, c_n$ by

$$\boldsymbol{b} = (b_1, \cdots, b_b, b_{b+1}, \cdots, b_{Nb})$$

The group of bits $b_{(i-1)b+1}, \cdots, b_{ib}$ is used to select the constellation point for the i-th transmit antenna, denoted by $c_i, i = 1,2,\ldots,N.$ The log-likelihood for the l-th element of $\boldsymbol{b}$, b_l, is given by

$$\Lambda(b_l) = \log\frac{\Pr[b_l = 1 \mid r_1, \cdots, r_M]}{\Pr[b_l = 0 \mid r_1, \cdots, r_M]} = \log\frac{\Pr[b_l = 1, r_1, \cdots, r_M]}{\Pr[b_l = 0, r_1, \cdots, r_M]}$$

$$= \log\frac{\sum_{\boldsymbol{b}:b_l=1}\Pr[r_1, \cdots, r_M, \boldsymbol{b}]}{\sum_{\boldsymbol{b}:b_l=0}\Pr[r_1, \cdots, r_M, \boldsymbol{b}]} = \log\frac{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=1}\Pr[r_1, \cdots, r_M, \boldsymbol{c}]}{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=0}\Pr[r_1, \cdots, r_M, \boldsymbol{c}]} \tag{6}$$

where $\boldsymbol{c} = (c_1, c_2, \cdots, c_N)$, and $f(\cdot)$ is the mapping from $\boldsymbol{b}$ to $\boldsymbol{c}$. Assuming that all constellation points are equally likely, we can write

$$\Lambda(b_l) = \log\frac{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=1}\Pr[r_1, \cdots, r_M \mid c_1, \cdots, c_N]}{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=0}\Pr[r_1, \cdots, r_M \mid c_1, \cdots, c_N]} = \log\frac{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=1}\prod_{j=1}^{M}\Pr[r_j \mid c_1, \cdots, c_N]}{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=0}\prod_{j=1}^{M}\Pr[r_j \mid c_1, \cdots, c_N]} \tag{7}$$

Substituting for the Gaussian noise statistics, we obtain the log-likelihood for the bit b_l

$$\Lambda(b_l) = \log\frac{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=1}\prod_{j=1}^{M}\exp\left(-\frac{\left|r_j - \sum_{i=1}^{N}\alpha_{i,j}c_i\right|^2}{N_0}\right)}{\sum_{\boldsymbol{c}:\boldsymbol{c}=f(\boldsymbol{b}),b_l=0}\prod_{j=1}^{M}\exp\left(-\frac{\left|r_j - \sum_{i=1}^{N}\alpha_{i,j}c_i\right|^2}{N_0}\right)} \tag{8}$$

We can use this log-likelihood to make the decision for every bit to detect the transmitted signal.

C. The Comparison of the Three Algorithms' Applicability

In ZF algorithm, in order to recover the transmitted signal on the k_i-th transmit antenna, the ZF criterion requires equation (4) to be satisfied. This requirement is satisfied by selecting the column vectors of $\boldsymbol{H}^{+}_{N\times M}$ as the weight vectors. Thus the orthogonality requirement in equation (4) is equivalent to

$$\boldsymbol{H}^{+}_{N\times M}\boldsymbol{H}_{M\times N} = \boldsymbol{I}_{N\times N}$$

Because the rank of matrix $\boldsymbol{I}_{N\times N}$ is

$$Rank(\boldsymbol{I}) = N$$

and

$$Rank(\boldsymbol{I}) \le \min[Rank(\boldsymbol{H}^{+}_{N\times M}), Rank(\boldsymbol{H}_{M\times N})] \quad ,$$

we can see that $\boldsymbol{H}_{M\times N}$ must be a matrix whose $Rank(\boldsymbol{H}_{M\times N}) = N$ and $M \ge N$. In other words, the ZF algorithm requires the number of receive antennas more than that of transmit antennas. However, in space-time codes scheme the transmitter usually has more than two antennas to obtain high transmitting spectral efficiency, which means the receiver also has more than two antennas. But because of the limit of half wavelength spacing to antenna array, it is impractical to build many antennas on a very small mobile receiver. The MMSE algorithm has similar limit as ZF does. Consequently the application of LST is primarily in packet data service so far and greatly limited under the mobile environment.

Equation (8) suggests that ML algorithm has no requirement with the number of receive antennas. Were there only one receive antenna ML algorithm could still decode correctly at the cost of the losing of the diversity gain. However, although ML decoding achieves the lowest error probability, computation complexity is very high. Generally the complexity of ML decoding is exponential with *N*. Consider space-time codes that encode $aN + \mathrm{O}(N)$ bits per n-tuple, where a is a positive constant. When an n-tuple $\boldsymbol{c}_\tau$ is transmitted, the log-likelihood of the received vector $\boldsymbol{r}$ is $|\boldsymbol{r} - \boldsymbol{H}\boldsymbol{c}_\tau|^2$. For a non-trivial $\boldsymbol{H}$, $|\boldsymbol{r} - \boldsymbol{H}\boldsymbol{c}_\tau|^2$ cannot be further reduced; thus an exhaustive search among the $2^{aN+\mathrm{O}(N)}$ possible choices of $\boldsymbol{c}_\tau$ requires a large amount of computation times. When there are many transmit antennas, ML decoding has an extremely high complexity. So ML decoding is especially fit for the system with few antennas rather than that with many antennas.

IV. SIMULATION RESULTS

We perform the simulations over both flat fading channel and quasi-static Rayleigh fading channel. In the quasi-static Rayleigh fading case, we use the Case1 model in 3GPP where two fading paths are assumed and the signal power of the second path is one tenth of that of the first. The system bit rate is 144Kb/s in Case1.

The performance of VLST codes is presented where turbo code with code rate *R=1/3* is involved. The QPSK constellation is used at each transmit antenna and the SNR is defined as

$$SNR = NE_s / N_0 \quad ,$$

where N is the number of transmit antennas, E_s the average energy of the signal constellation at each transmit antenna .For the normalized energy, E_s =1.

A. The Performance Comparison over Flat Fading Channel

Fig.2 depicts the performance comparison of the three algorithms. We suppose flat fading and the system has two transmit and two receive antennas. It is obvious that ML algorithm, as the optimum decoding method, has much better performance than the other two. This advantage is obtained at the cost of high complexity.

We can also find that MMSE has a gain about 2dB compared to ZF. The phenomena can be explained as following. The coding processes of ZF and MMSE are performed layer by layer and then the detection process of each layer has much influence on the following layers. In this way the influence of additive noise will be enhanced layer by layer. In ZF algorithm weight vectors are required to be orthogonal with the columns of the channel matrix to eliminate the fading corruptions and some useful information is lost without considering the influence of additive noise. However, MMSE algorithm chooses the weight vectors by MMSE criterion. The influence of the additive noise is considered as well as the channel matrix. Hence the propagation effect of additive noise is weakened and the system performance is improved.

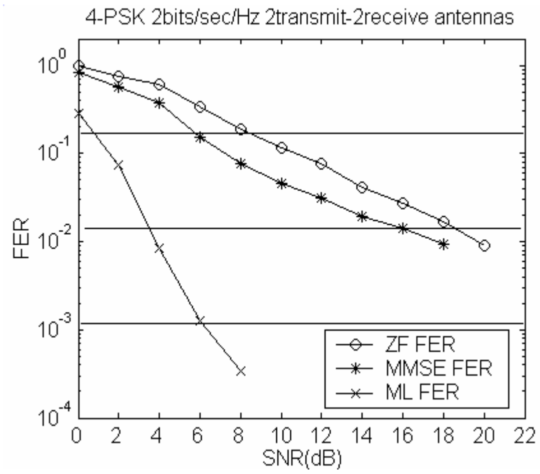

Fig.2. Performance comparison of three algorithms over flat fading channel.

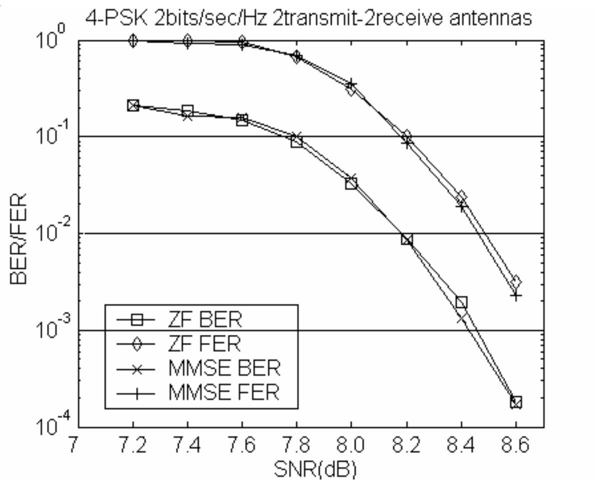

Fig.3. Performance comparison of ZF and MMSE over Case1 Channel

B. The Performance Comparison over Quasi-Static Rayleigh Fading Channel

Fig.3 shows the performance comparison of ZF and MMSE over the 3GPP Case1 channel. The bit rate is 144Kb/s and the system has two transmit and two receive antennas. In Fig.3 the performance curves of ZF and MMSE are nearly superposed. The gain of MMSE compared to ZF cannot be obtained over the Rayleigh fading channel. The main reason is that under 3GPP environment the corruption of the channel matrix to the transmitted signal is much more than the influence of the additive noise. We can use Rake receiver to separate the signals received from different paths, but the scramble codes are not ideal, which results in that the interference between the fading paths cannot be eliminated thoroughly. Thus MMSE algorithm's advantage doesn't exist.

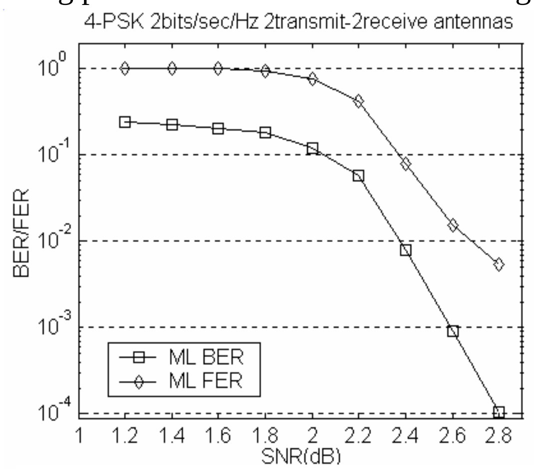

Fig.4. Performance of ML decoding with 2 receive antennas over Case1 channel

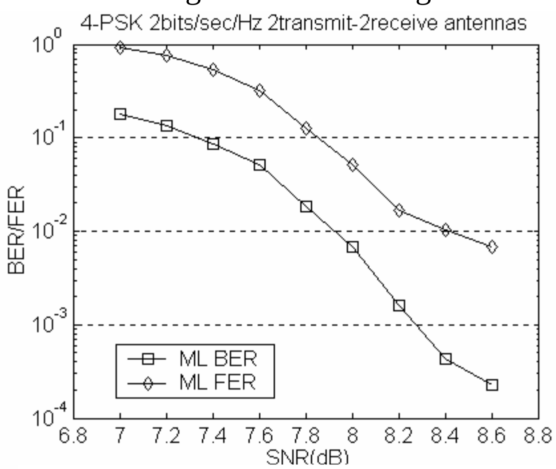

Fig.5. Performance of ML decoding with only 1 receive antennas over Case1 channel

C. The Performance of ML Algorithm under the Mobile Environment

Fig.4 and Fig.5 present the performance of ML decoding over Rayleigh fading channel. We also assume the 3GPP Case1 channel. The system in Fig.4 has two transmit and two receive antennas and in Fig.5 only one receive antenna. Compared with Fig.3, Fig.4 suggests that ML algorithm has much better anti-Rayleigh-fading ability. Over Case1 channel the gain of ML is 5.5dB compared to ZF and MMSE algorithms at a FER. of 10^{-2}.

Comparing Fig.4 with Fig.5 we can find that the system with two receive antennas has 5.7 dB gain compared to the system with only one receive antenna. The reason is that the receiver in Fig.5 lacks the diversity gain.

V. CONCLUSION

From the above discussion we can see that MMSE algorithm is fit for the VLST system whose transmitter and receiver are both stationary under the flat fading environment. In this case a VLST system with many antennas, such as four or eight antennas, can achieve very high bit rate and spectral efficiency by using MMSE decoding algorithm. On the contrary, ML algorithm has a high complexity and it is unfit for the VLST system with so many antennas.

When the receiver is small and mobile and under the Rayleigh fading environment, ML decoding, however, has better performance and applicability than ZF and MMSE decoding because the former does not have the limit that the number of receive antennas is greater than that of the transmit.

In the engineering application the decoding algorithm must be selected according to the practical environment. For example, in the high-speed wireless packet access network we can use MMSE decoding for VLST system to provide the high-speed access service for user devices. However, in the mobile communication network, the VLST receiver with very few antennas, such as only one antenna, will provide high-speed data service by using ML decoding algorithm.

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